

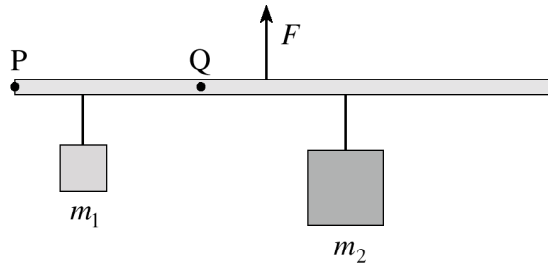
Ultimate Equilibrium Review Assignment

Key Formulae:

$$\tau = F_{\perp} d$$

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1.

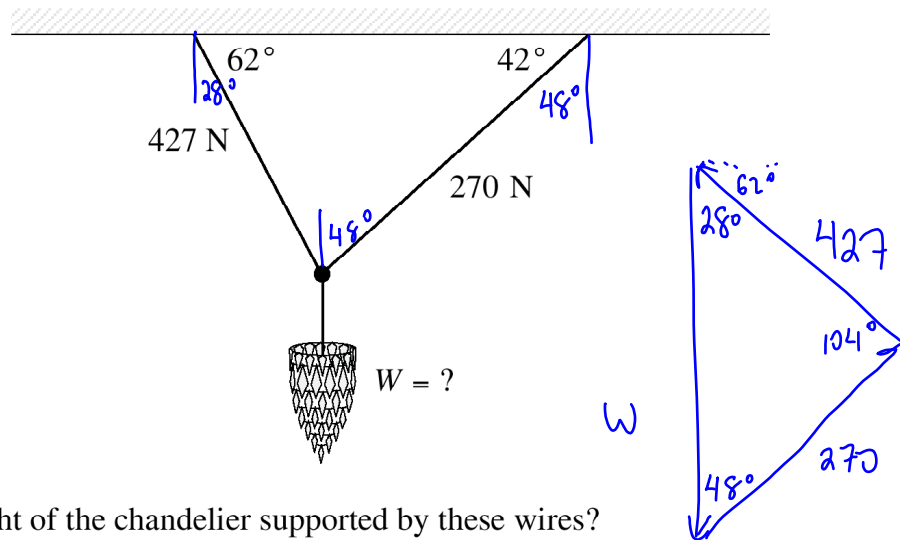
A beam holding two masses is in static equilibrium.
 $\therefore \sum \tau = 0$ at any location


Compare the sum of the torques about point P to the sum of the torques about point Q.

- A. The sum of the torques is the same about both point P and point Q.
- B. You need to know the mass of the beam to compare the sum of the torques.
- C. The sum of the torques about point P is less than the sum of the torques about point Q.
- D. The sum of the torques about point P is greater than the sum of the torques about point Q.

2.

In the diagram below, the tension in each wire is shown.



What is the weight of the chandelier supported by these wires?

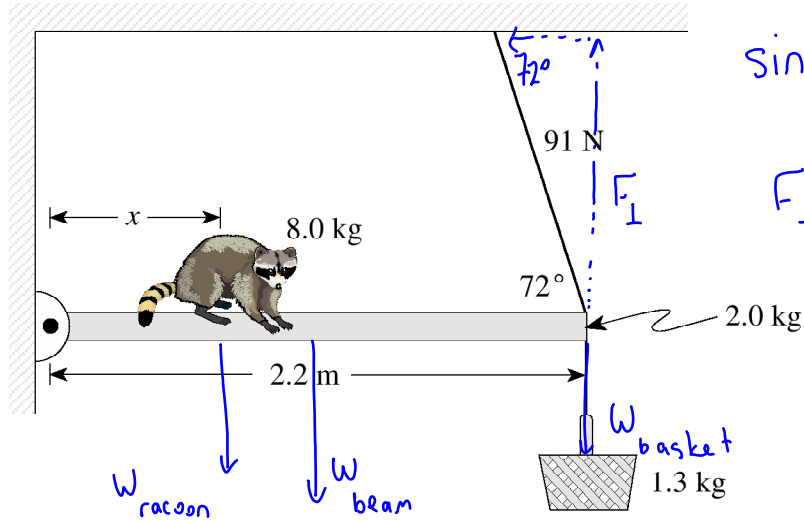
- A. 300 N
- B. 510 N
- C. 560 N
- D. 620 N

$$\frac{\sin 104}{W} = \frac{\sin 28}{270}$$

$$W = \frac{270 \sin 104}{\sin 28} = 558 \text{ N}$$

3.

- A hungry 8.0 kg raccoon walks out on a 2.0 kg, 2.2 m long uniform beam in an attempt to reach a 1.3 kg food basket hanging at the end. A cord that can withstand 91 N is used to support the beam at the end as shown.



What is the maximum distance, x , the raccoon can walk out onto the beam before the cord breaks?

- A. 1.8 m
- B. 1.9 m
- C. 2.0 m
- D. 2.2 m

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$W_{\text{raccoon}} \cdot x + W_{\text{beam}} (1.1) + W_{\text{basket}} \times 2.2 = F \times 2.2$$

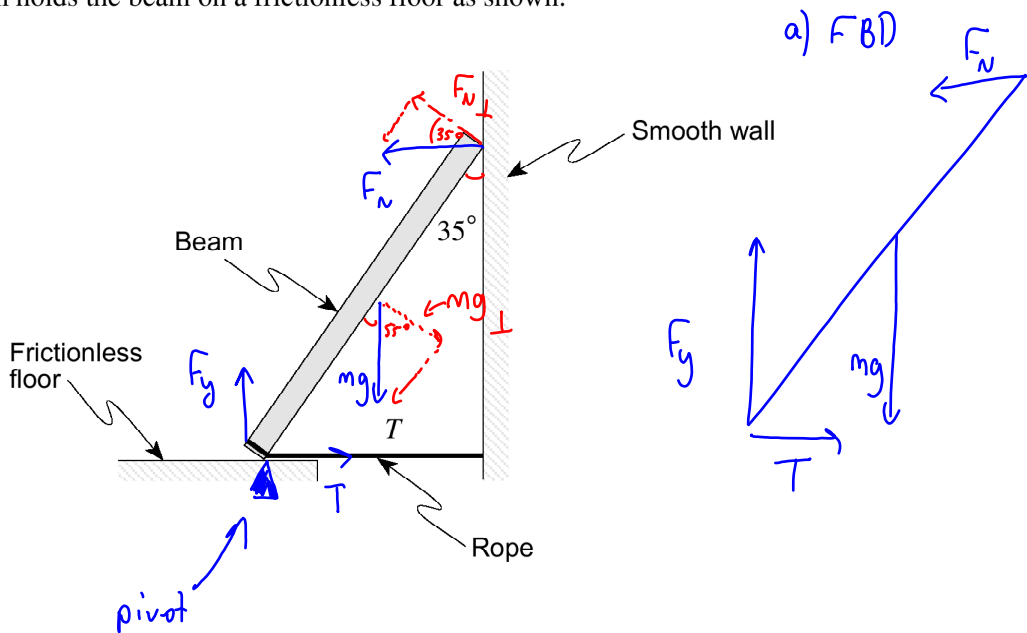
$$8(9.8)x + 2(9.8)(1.1) + 1.3(9.8)(2.2) = 91 \sin 72^\circ \times 2.2$$

$$x = \frac{91(\sin 72^\circ)(2.2) - 2(9.8)(1.1) - 1.3(9.8)(2.2)}{8(9.8)}$$

$$x = 1.796 \text{ m}$$

4.

A 24 kg beam of length 2.4 m leans against a smooth wall. A horizontal rope tied to the wall and the beam holds the beam on a frictionless floor as shown.



a) Draw a labelled free-body diagram for the forces acting on the beam. (2 marks)

b) What is the tension in the rope? (5 marks)

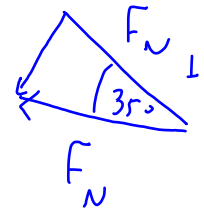
From FBD, $T = F_N$ ∴ Find F_N

$$\sum \tau_{cw} =$$

$$F_N \perp (2.4) - mg \perp (1.2)$$

$$F_N \perp (2.4) = mg \cos(55) (1.2)$$

$$F_N \perp = \frac{(24)(9.8) \cos(55) (1.2)}{2.4} = 67.45$$



$$\cos 35^\circ = \frac{F_N \perp}{F_N}$$

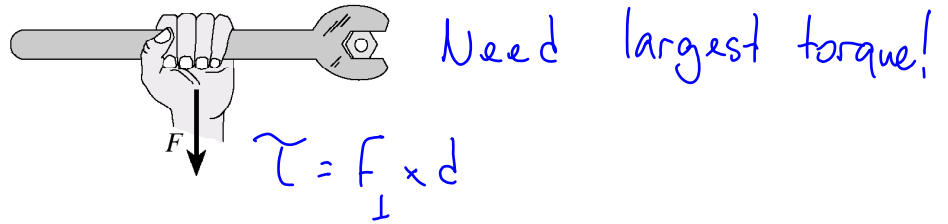
$$F_N = \frac{F_N \perp}{\cos 35} = 82 \text{ N}$$

$$F_N = 82 \text{ N} = \text{Tension}$$

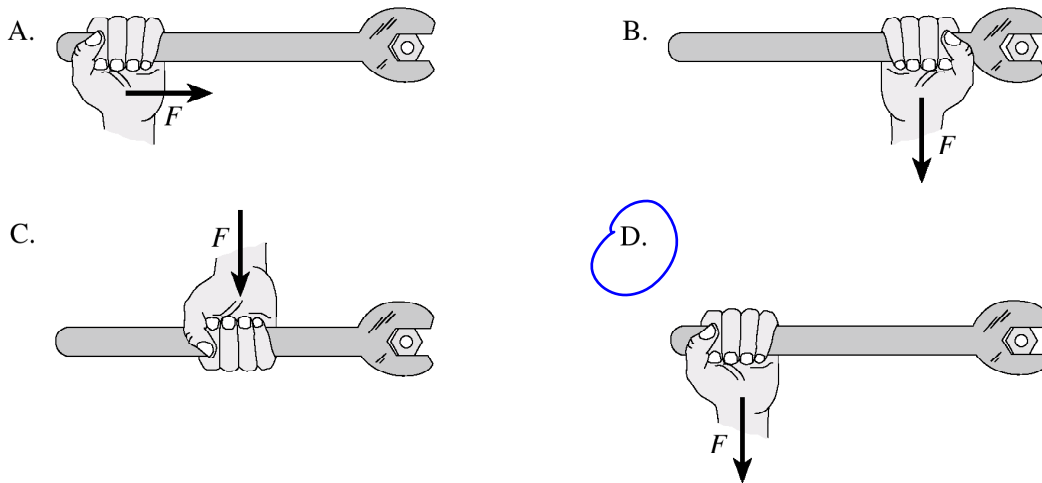
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5.

A student uses a wrench to loosen a very tight nut in the position shown.

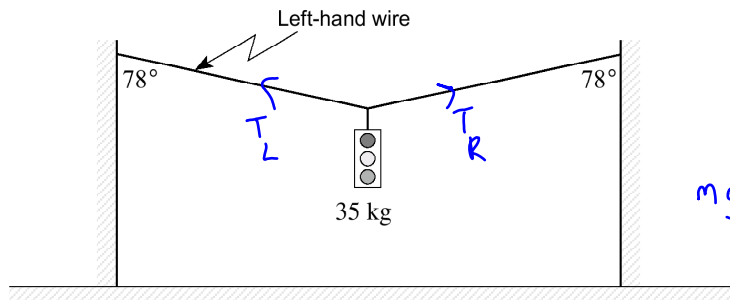


In which of the following would the student have more success at loosening the nut?

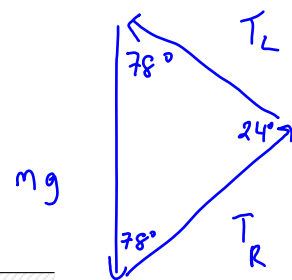


6.

A 35 kg traffic light is suspended by two wires as shown.



Solution 1:

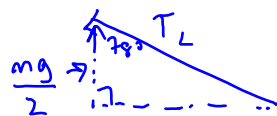


$$\frac{\sin 24}{(35)(9.8)} = \frac{\sin 78}{T_L}$$

What is the tension in the left-hand wire?

- A. 84 N
- B. 410 N
- C. 820 N
- D. 1 600 N

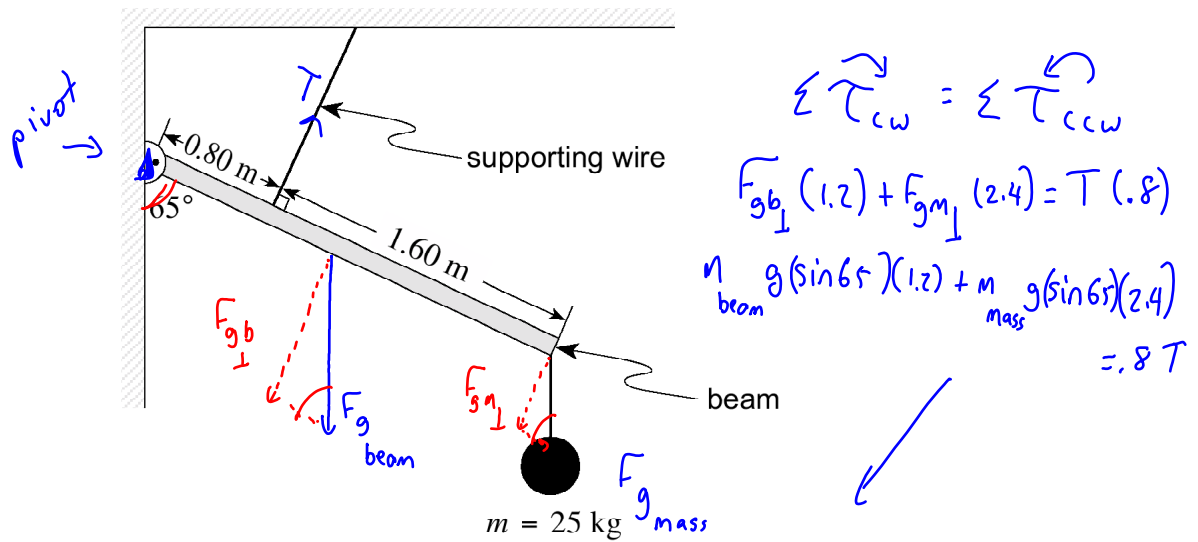
Solution 2: since angles are =, each cable supports $\frac{mg}{2}$



$$\cos 78^\circ = \frac{\frac{mg}{2}}{T_L}$$

7.

A 15 kg uniform beam 2.40 m long is suspended from a wall and a ceiling as shown.



What is the tension in the supporting wire?

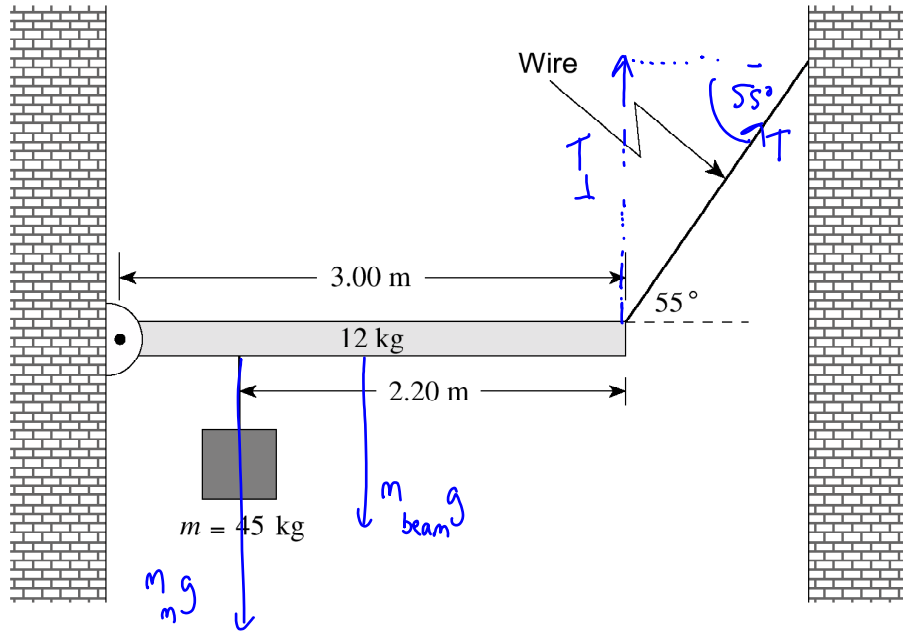
- A. 670 N
- B. 740 N
- C. 870 N
- D. 960 N

$$T = \frac{(15)(9.8)(\sin 65^\circ)(1.2) + 25(9.8)(\sin 65^\circ)(2.4)}{.8}$$

$$T = 866 \text{ N}$$

8.

A uniform 12 kg beam of length 3.00 m holding a 45 kg mass is attached by a wire to a wall as shown.



What is the tension in the wire?

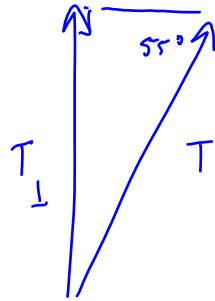
(7 marks)

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$m_m g (.8) + m_b g (1.5) = T_{\perp} (3)$$

$$T_{\perp} = \frac{(45)(9.8)(.8) + 12(9.8)(1.5)}{3}$$

$$T_{\perp} = 1764 \text{ N} \leftarrow \text{but we want } T \rightarrow$$



$$\sin 55^\circ = \frac{T_{\perp}}{T}$$

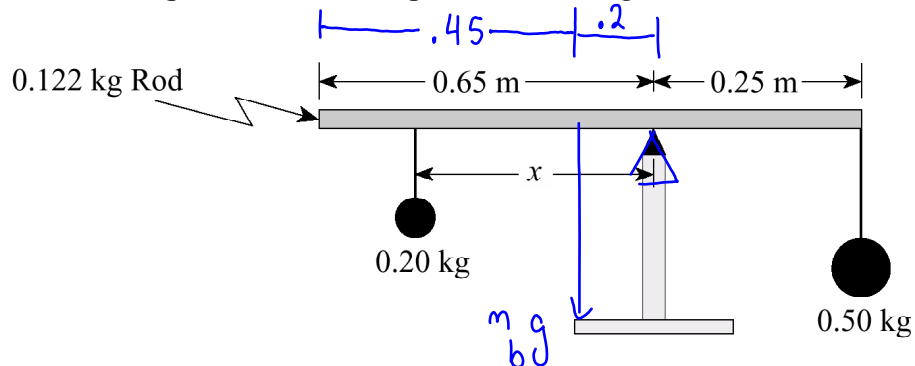
$$T = \frac{T_{\perp}}{\sin 55^\circ}$$

$$T = 2.15 \times 10^3 \text{ N}$$

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9.

A uniform 0.122 kg rod of 0.90 m length is used to suspend two masses as shown below.



At what distance x should the 0.20 kg mass be placed to achieve static equilibrium?

- A. 0.30 m
- B. 0.50 m
- C. 0.63 m
- D. 0.75 m

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

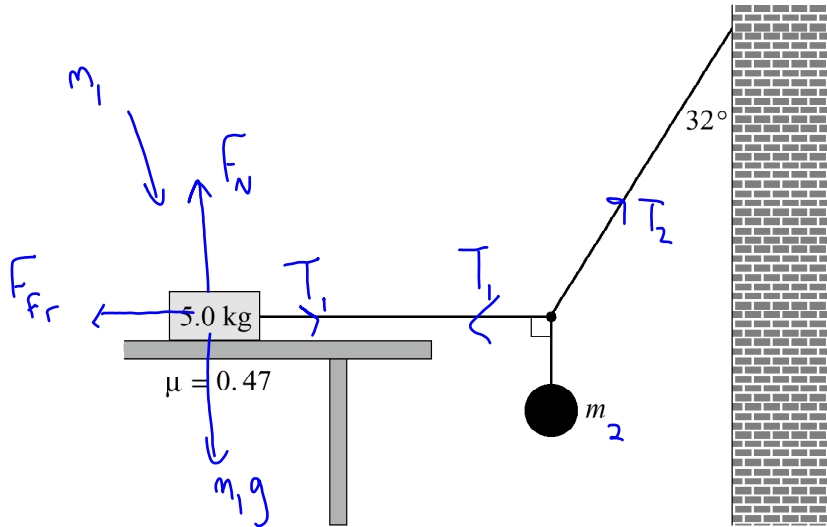
$$(0.5)g(0.25) = m_b g(0.2) + 0.2(g)x$$

$$x = \frac{0.5(g)(0.25) - m_b g(0.2)}{0.2g} = \frac{(0.5)(9.8)(0.25) - (0.122)(9.8)(0.2)}{(0.2)(9.8)}$$

$$x = 0.503 \text{ m}$$

10.

An object of mass, m , is suspended by two cords connected to a wall and to a 5.0 kg block resting on a table as shown.

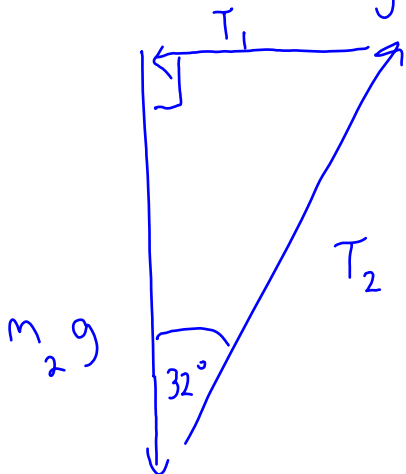


A coefficient of friction of 0.47 exists between the 5.0 kg block and the table. What is the maximum mass, m , that can be hung from the cords before the 5.0 kg block begins to move?

(7 marks)

We want $F_{fr} = T_1$

$$\mu m_1 g = T_1 \rightarrow T_1 = (0.47)(5)(9.8) = 23.03 \text{ N}$$



$$\tan 32 = \frac{T_1}{m_2 g}$$

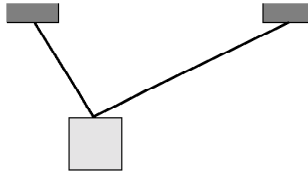
$$m_2 = \frac{T_1}{g \tan 32} = \frac{23.03}{(9.8)(\tan 32)} = 3.76 \text{ kg}$$

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11.

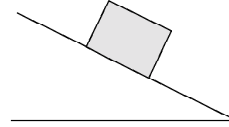
Which of the four problems shown requires the application of torque? *need a beam!*

A.



What is the tension in the supporting cables?

B.



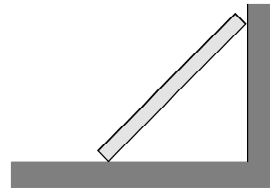
What is the friction force acting on the block?

C.



What is the acceleration of the puck?

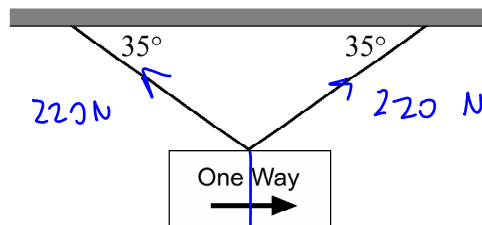
D.



What force does the wall exert on the board?

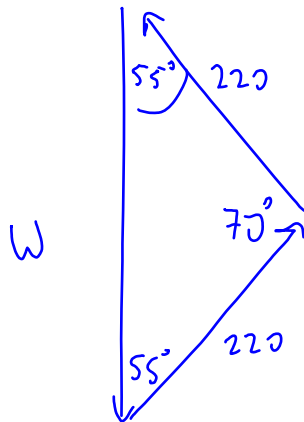
12.

A traffic sign hangs from two cables as shown.



If the tension in each cable is 220 N, what is the weight of the sign?

- A. 130 N
- B. 250 N
- C. 360 N
- D. 440 N

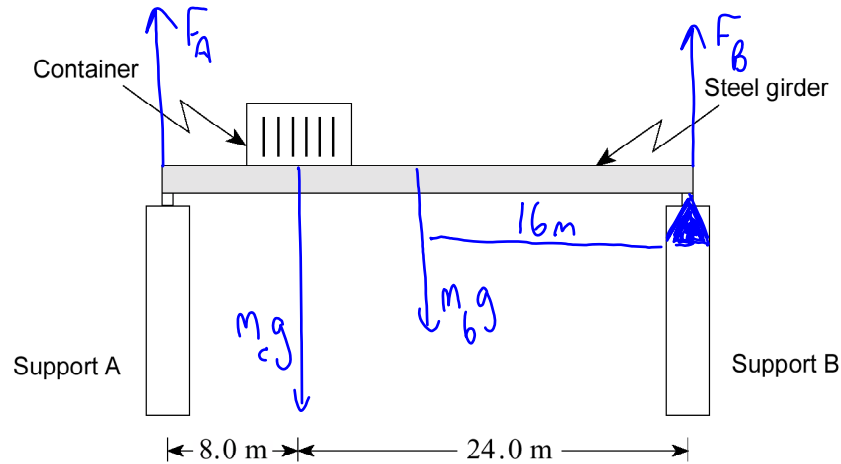


$$\frac{\sin 70}{W} = \frac{\sin 55}{220}$$

$$W = \frac{220 \sin 70}{\sin 55} = 252 \text{ N}$$

13.

A uniform 1 200 kg steel girder is supported horizontally at its endpoints as shown in the diagram.



What are the upward forces at the girder end points when it is bearing a 3 700 kg shipping container 8.0 m from support A? (7 marks)

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$F_A (32) = m_c g (24) + m_b g (16)$$

$$F_A = \frac{m_c g (24) + m_b g (16)}{32} = \frac{(3700)(9.8)(24) + (1200)(9.8)(16)}{32}$$

$$F_A = 33075 \text{ N} = 3.3 \times 10^4 \text{ N}$$

$$F_A + F_B = m_c g + m_b g$$

$$F_B = m_c g + m_b g - F_A = (3700)(9.8) + (1200)(9.8) - 33075$$

$$F_B = 14945 \text{ N} = 1.49 \times 10^4 \text{ N}$$

14.

In your summer job with the Ministry of Transportation and Highways your supervisor has told you that street signs should no longer be suspended as shown in Diagram A. In order to save money, he would prefer a shorter, perfectly horizontal cable, as shown in Diagram B.

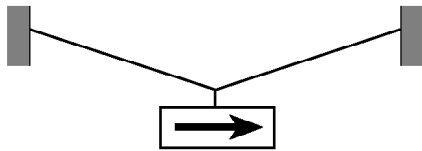


Diagram A

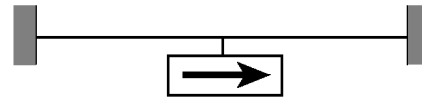


Diagram B

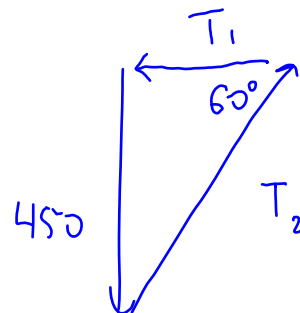
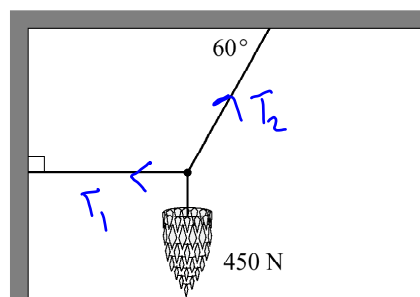
Using principles of physics, argue that the situation in Diagram B is not reasonable. (4 marks)

see answer key at end of assignment

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15.

A 450 N chandelier is supported by three cables as shown in the diagram.



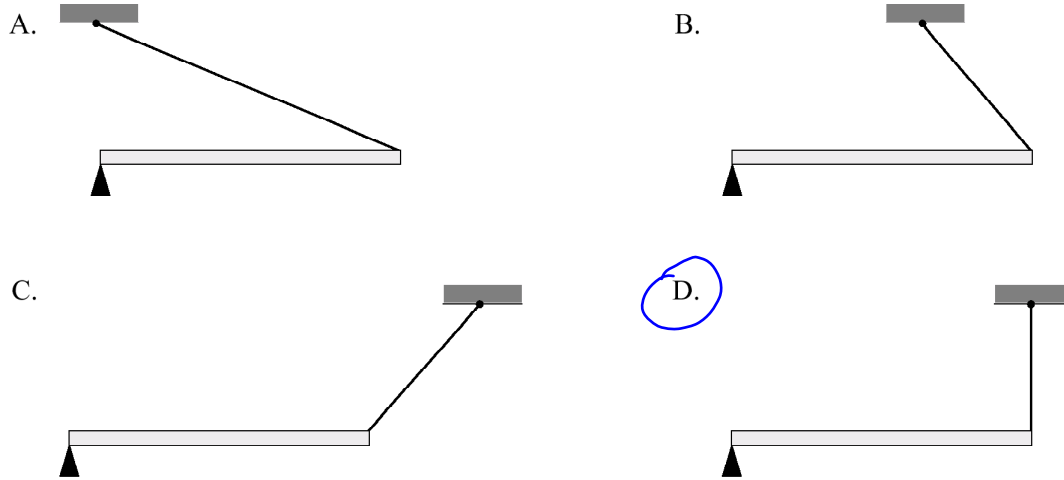
What is the tension in the horizontal cable?

- A. 46 N
- B. 260 N
- C. 450 N
- D. 520 N

$$\tan 60^\circ = \frac{450}{T_1} \rightarrow T_1 = \frac{450}{\tan 60^\circ}$$

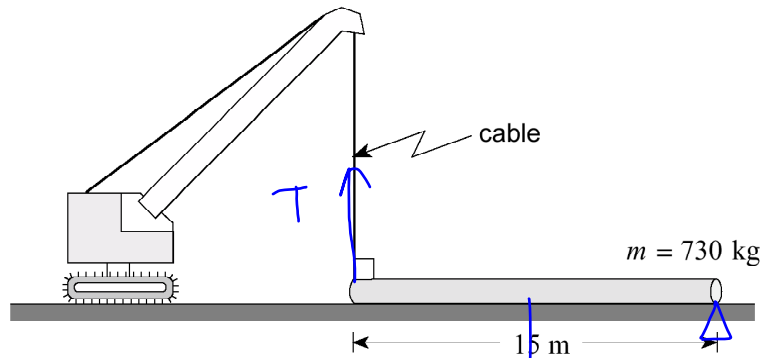
16.

A beam is to be kept horizontal by a cord. In which of the four situations shown below will the tension in the cord be least?



17.

A crane is used to lift one end of a uniform 15 m long pipe with a mass of 730 kg as shown in the diagram below.



What is the minimum force of tension in the crane cable to just lift the end of the pipe off the ground?

- A. 3.7×10^2 N
- B. 4.8×10^2 N
- C. 3.6×10^3 N
- D. 7.2×10^3 N

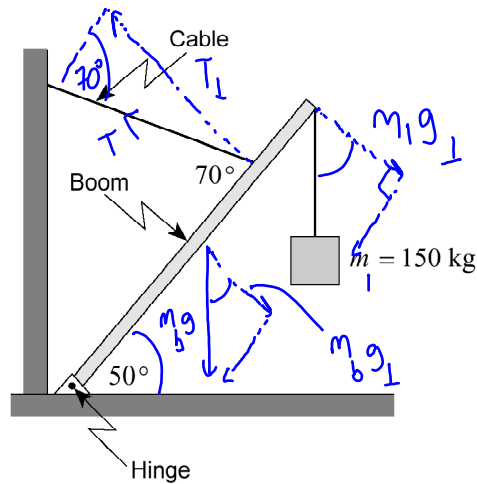
$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$T(15) = m_b g(7.5)$$

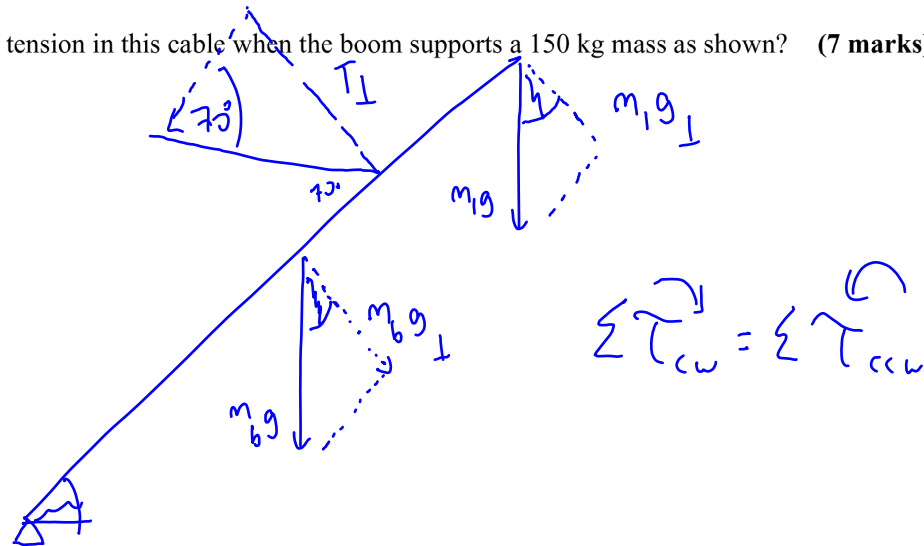
$$T = \frac{m_b g(7.5)}{15} = 3577 \text{ N}$$

18.

A uniform 6.0 m-long boom has a mass of 55 kg. It is kept in position by a restraining cable attached three-quarters of the way along the boom.



What is the tension in this cable when the boom supports a 150 kg mass as shown? (7 marks)



$$m_b g_{\perp} (3) + m_1 g_{\perp} (6) = T_{\perp} (4.5)$$

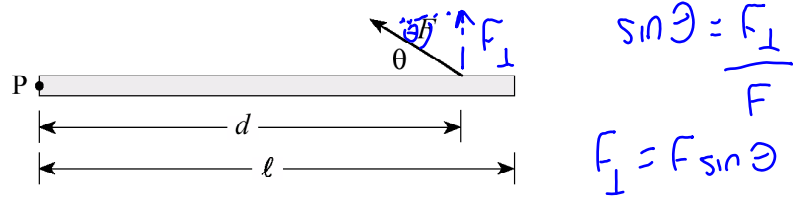
$$\frac{m_b g (\cos 50) (3) + m_1 g (\cos 50) (6)}{4.5} = T_{\perp} \rightarrow T_{\perp} = 1490.8$$

$$\sin 70^{\circ} = \frac{T_{\perp}}{T} \rightarrow T = \frac{T_{\perp}}{\sin 70^{\circ}} = 1590 \text{ N} = 1.59 \times 10^3 \text{ N}$$

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19.

A force F is applied to a uniform horizontal beam as shown in the diagram below.



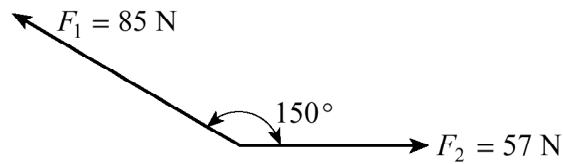
Which of the following is a correct expression for the torque on the beam about pivot point P due to this force?

- A. $F \sin \theta \cdot d$
- B. $F \sin \theta \cdot d/l$
- C. $F \cos \theta \cdot d$
- D. $F \cos \theta \cdot d/l$

$$\tau = F \cdot d = F \sin \theta \cdot d$$

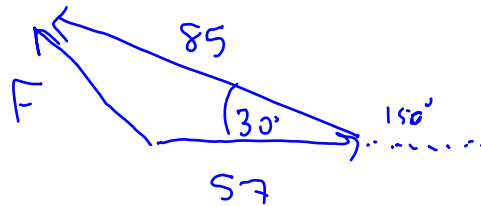
20.

What is the magnitude of the sum of the two forces shown in the diagram below?



add tip to tail

- A. 46 N
- B. 102 N
- C. 137 N
- D. 142 N



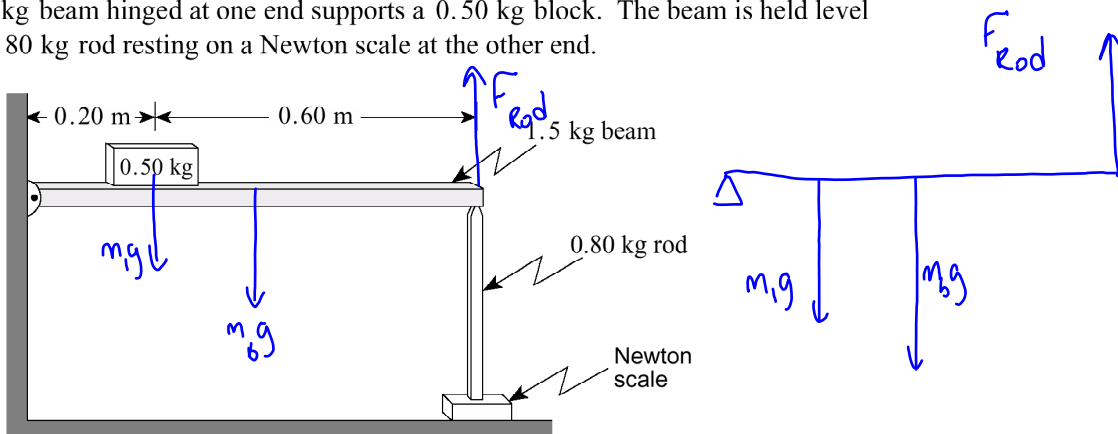
$$F^2 = 57^2 + 85^2 - 2(57)(85)\cos 30$$

$$F = 45.6 \text{ N}$$

Tough question!

21.

A uniform 1.5 kg beam hinged at one end supports a 0.50 kg block. The beam is held level by a vertical 0.80 kg rod resting on a Newton scale at the other end.



What is the reading on the scale?

- A. 8.6 N
- B. 9.1 N
- C. 16 N
- D. 27 N

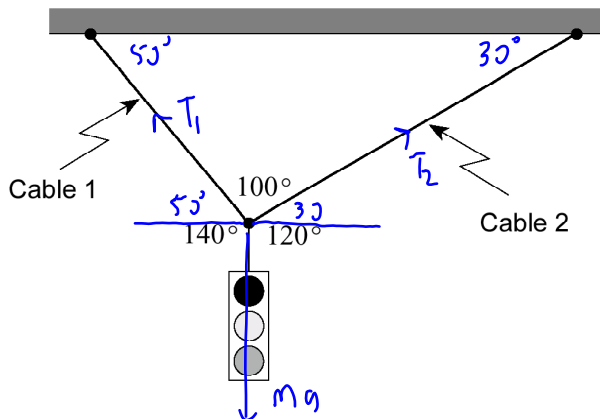
$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$m_1 g (0.2) + m_2 g (0.8) = F_{Rod} (0.8) \rightarrow F_{Rod} = 8.575$$

$$\text{Scale} = F_{Rod} + m_{Rod} g = 8.575 + 0.8(9.8) = 16.4 \text{ N}$$

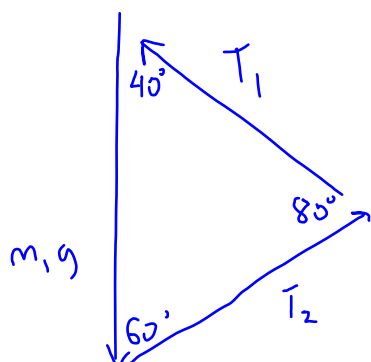
22.

A 35 kg traffic light is suspended from two cables as shown in the diagram.



What is the tension in each of these cables?

(7 marks)



$$\frac{\sin 80^\circ}{mg} = \frac{\sin 60^\circ}{T_1}$$

$$T_1 = \frac{mg \sin 60^\circ}{\sin 80^\circ} = 302 \text{ N}$$

$$\frac{\sin 80^\circ}{mg} = \frac{\sin 40^\circ}{T_2} \rightarrow T_2 = \frac{mg \sin 40^\circ}{\sin 80^\circ} = 224 \text{ N}$$

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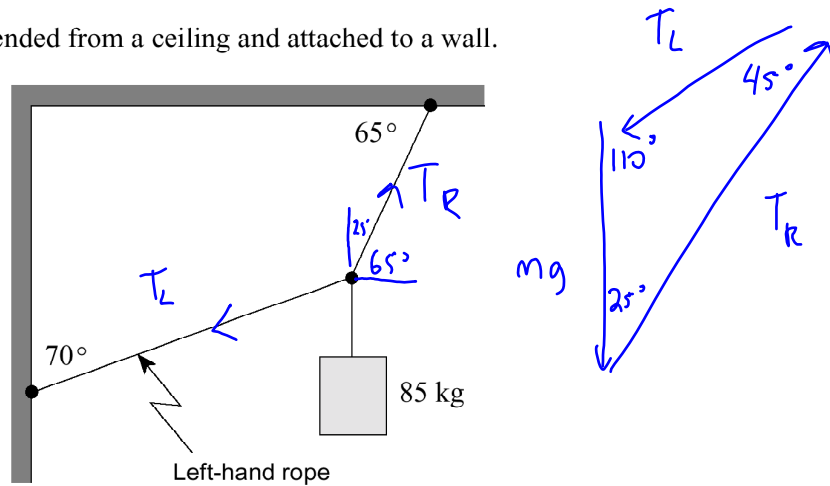
23.

The unit for torque is

- A. J
- B. N·m
- C. N·s
- D. kg·m/s

24.

An 85 kg object is suspended from a ceiling and attached to a wall.



What is the tension in the left-hand rope?

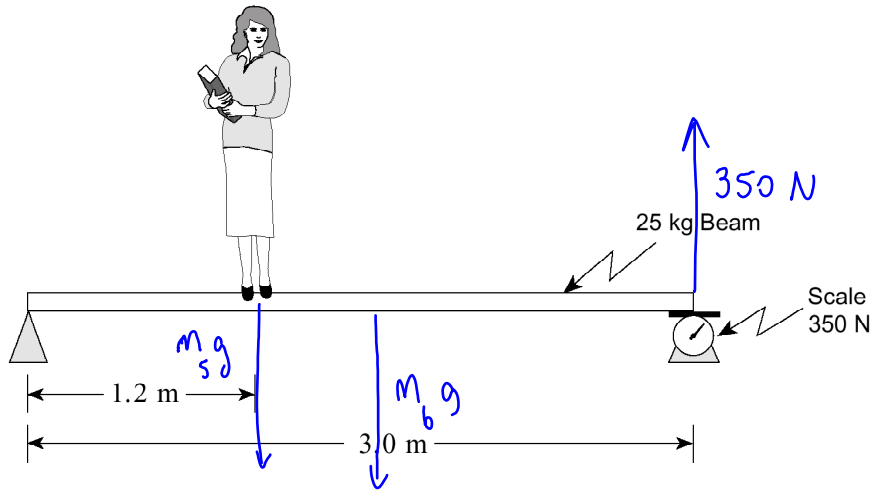
- A. 280 N
- B. 350 N
- C. 500 N
- D. 1100 N

$$\frac{\sin 25}{T_L} = \frac{\sin 45}{mg}$$

$$T_L = \frac{mg \sin 25}{\sin 45} = 498 \text{ N}$$

25.

A student stands on a uniform 25 kg beam. The scale on the right end reads 350 N.



What is the mass of the student?

- A. 45 kg
- B. 54 kg
- C. 58 kg
- D. 89 kg

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

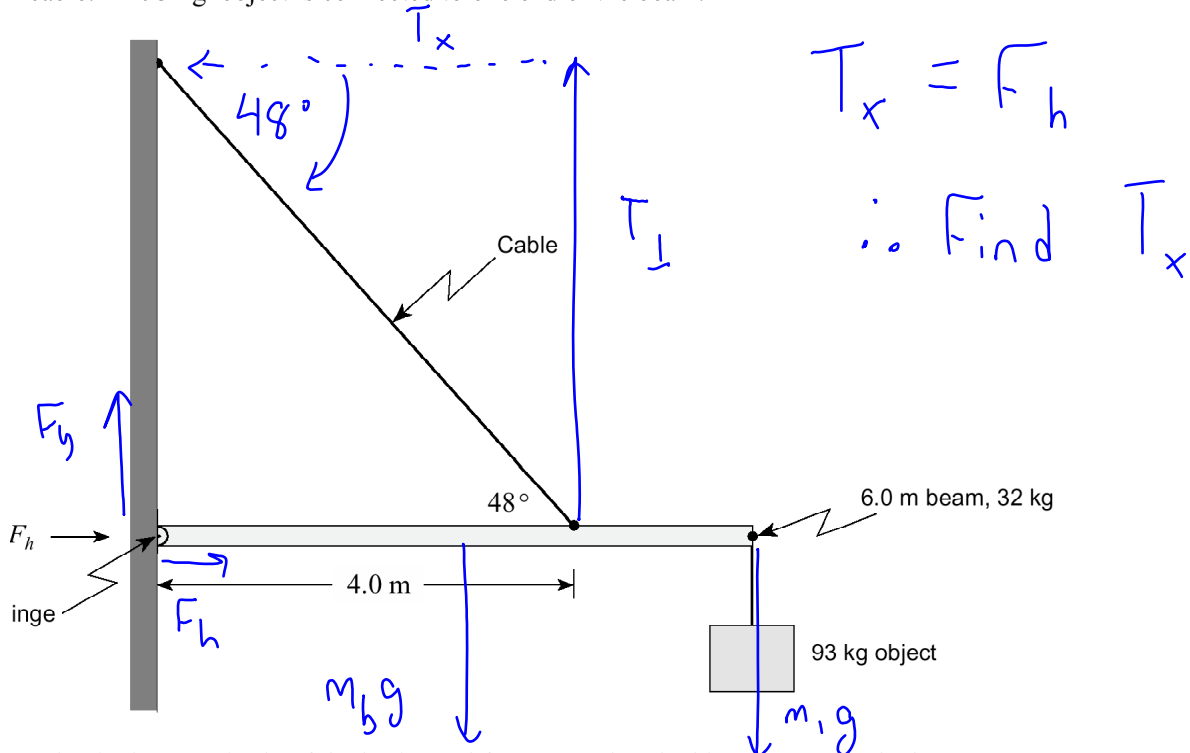
$$m_s g (1.2) + m_b g (1.5) = 350 (3)$$

$$m_s g (1.2) = (350)(3) - m_b g (1.5)$$

$$m_s = \frac{(350)(3) - 25(9.8)(1.5)}{(9.8)(1.2)} = 58 \text{ kg}$$

26.

A 6.0 m uniform beam of mass 32 kg is suspended horizontally by a hinged end and a cable. A 93 kg object is connected to one end of the beam.

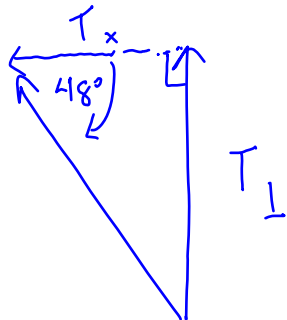


What is the magnitude of the horizontal force F_h that the hinge exerts on the beam? (7 marks)

$$\sum \tau_{ccw} = \sum \tau_{cw}$$

$$m_b g (3) + m_o g (6) = T_{\perp} (4)$$

$$\frac{(32)(9.8)(3) + 93(9.8)(6)}{4} = T_{\perp} \rightarrow T_{\perp} = 1602.3 \text{ N}$$



$$\tan 48^\circ = \frac{T_{\perp}}{T_x}$$

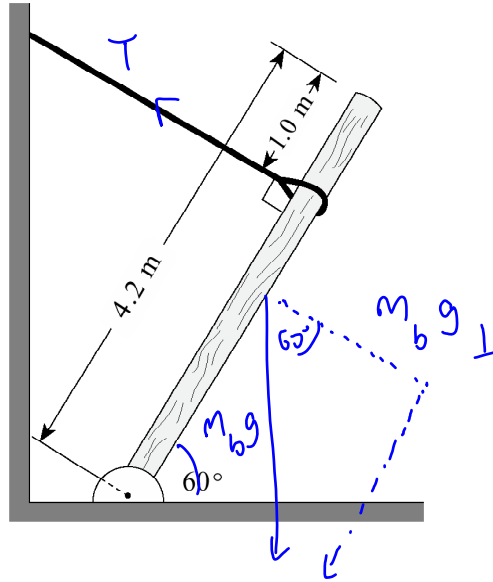
$$T_x = \frac{T_{\perp}}{\tan 48} = \frac{1602.3}{\tan 48}$$

$$T_x = F_h = 1442.7 \text{ N} = 1.44 \times 10^3 \text{ N}$$

9906

27.

A 4.2 m long uniform post is supported by a cable having a tension of 1 700 N. What is the mass of this post?



- A. 160 kg
- B. 260 kg
- C. 300 kg
- D. 530 kg

$$m_b g_{\perp} (2.1) = T (3.2)$$

$$mg(\cos 60)(2.1) = (1700)(3.2)$$

$$m = \frac{(1700)(3.2)}{g(\cos 60)2.1}$$

$$m = 529 \text{ kg}$$

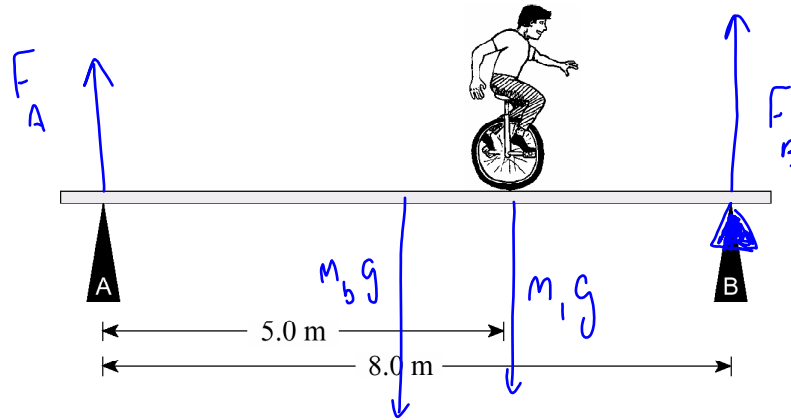
28.

A body is in static equilibrium when

- A. $\Sigma\tau = 0$ only.
- B. $\Sigma F = 0$ only.
- C. $\Sigma F = 0$ and $\Sigma\tau = 0$.
- D. $\Sigma F = 0$ and $\Sigma\tau \neq 0$.

29.

A circus performer on a unicycle of total mass 55 kg rides across a uniform 30 kg beam. The supports are placed equal distances from the ends of the beam.



a) When he is at the position shown, determine the forces exerted by the supports on the beam. (5 marks)

$$\sum \tau_{ccw} = \sum \tau_{cw}$$

$$F_A (8) = m_b g (4) + m_1 g (3)$$

$$F_A = \frac{(30)(9.8)(4) + (55)(9.8)(3)}{8} = 349 \text{ N}$$

$$\left. \begin{aligned} F_A + F_B &= m_b g + m_1 g \\ F_B &= m_b g + m_1 g - F_A \\ F_B &= 484 \text{ N} \end{aligned} \right\}$$

b) As the performer moves toward the right the force exerted by support B will

- remain the same.
 increase.
 decrease.

(Check one response.)

(1 mark)

c) Using principles of physics, explain your answer to b).

(3 marks)

Put pivot at A.

$$\sum \tau_{ccw} = \sum \tau_{cw}$$

$$m_b g (4) + m_1 g (d) = F_B (8)$$

As performer moved right, "d" increases, and F_B increases.

9901

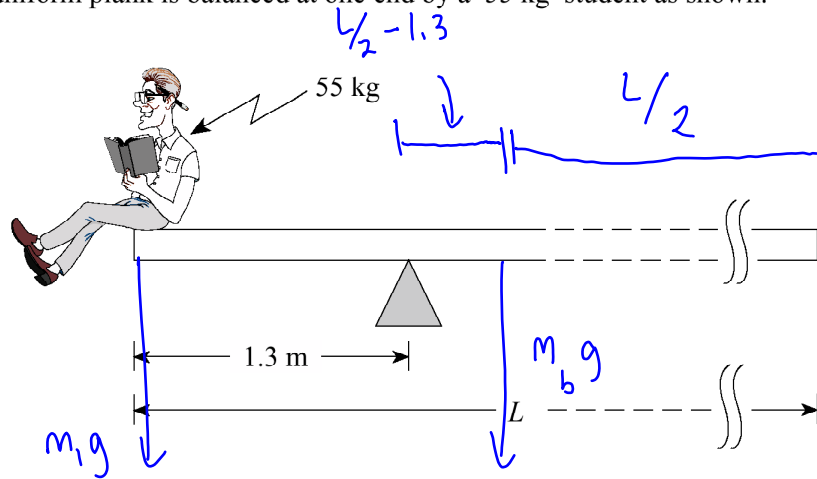
30.

A body is in rotational equilibrium when

- A. $\Sigma \tau = 0$
- B. $\Sigma F = 0$
- C. $\Sigma p = 0$
- D. $\Sigma E_k = 0$

31.

A 35 kg uniform plank is balanced at one end by a 55 kg student as shown.



What is the overall length of this plank?

- A. 2.6 m
- B. 3.3 m
- C. 5.4 m
- D. 6.7 m

$$\Sigma \tau_{\text{cw}} = \Sigma \tau_{\text{ccw}}$$
$$m_b g \left(\frac{L}{2} - 1.3 \right) = m_1 g (1.3)$$

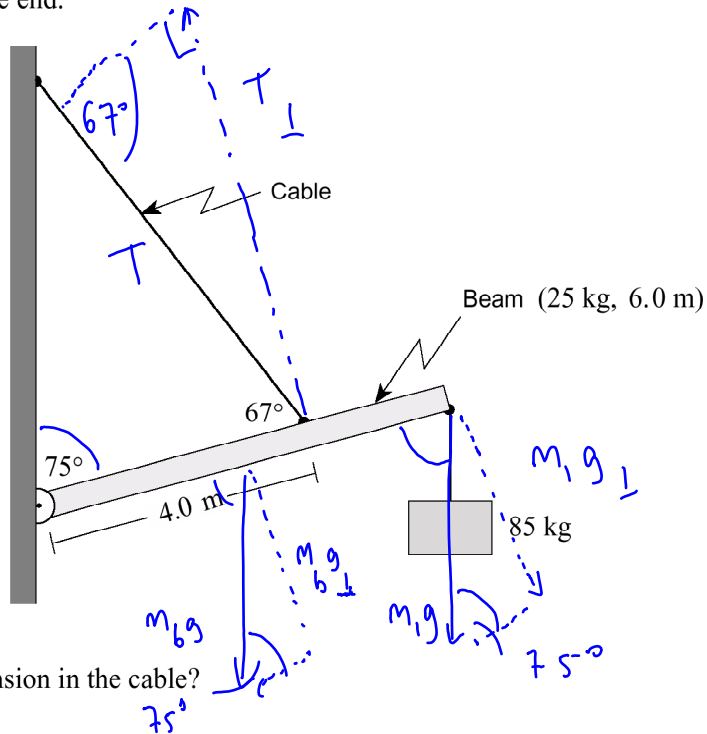
$$\frac{L}{2} - 1.3 = \frac{m_1 g (1.3)}{m_b g}$$

$$\frac{L}{2} = \frac{m_1 g (1.3)}{m_b g} + 1.3 \rightarrow \frac{L}{2} = \frac{(55)(9.8)(1.3)}{(35)(9.8)} + 1.3$$

$$\frac{L}{2} = 3.343 \rightarrow L = 6.69 \text{ m}$$

32.

A 6.0 m uniform beam of mass 25 kg is suspended by a cable as shown. An 85 kg object hangs from one end.



What is the tension in the cable?

(7 marks)

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$m_b g_{\perp} (3) + m_o g_{\perp} (6) = T_{\perp} (4)$$

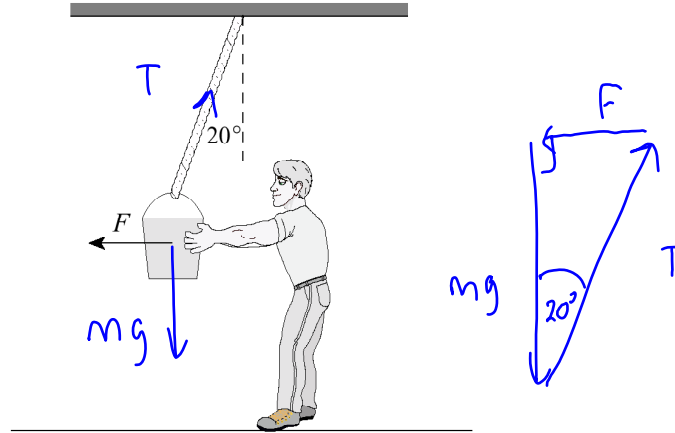
$$\frac{(25)(9.8)(\sin 75)(3) + (85)(9.8)(\sin 75)(6)}{4} = T_{\perp} \rightarrow T_{\perp} = 1384.4 \text{ N}$$

$$\sin 67 = \frac{T_{\perp}}{T} \rightarrow T = \frac{T_{\perp}}{\sin 67} = 1.5 \times 10^3 \text{ N}$$

9808

33.

Peter exerts a horizontal force F on a 12 kg bucket of concrete so that the supporting rope makes an angle of 20° with the vertical.



a) Find the tension force in the supporting rope.

(5 marks)

$$\cos 20^\circ = \frac{mg}{T} \rightarrow T = \frac{mg}{\cos 20^\circ} = \frac{12(9.8)}{\cos 20^\circ} = 125 \text{ N}$$

b) Peter now exerts a new force which causes the rope to make a greater angle with the vertical. How will the tension force in the supporting rope change?

- The tension force will increase.
 The tension force will decrease.
 The tension force will remain the same.

(Check one response.)

(1 mark)

c) Using principles of physics, explain your answer to b).

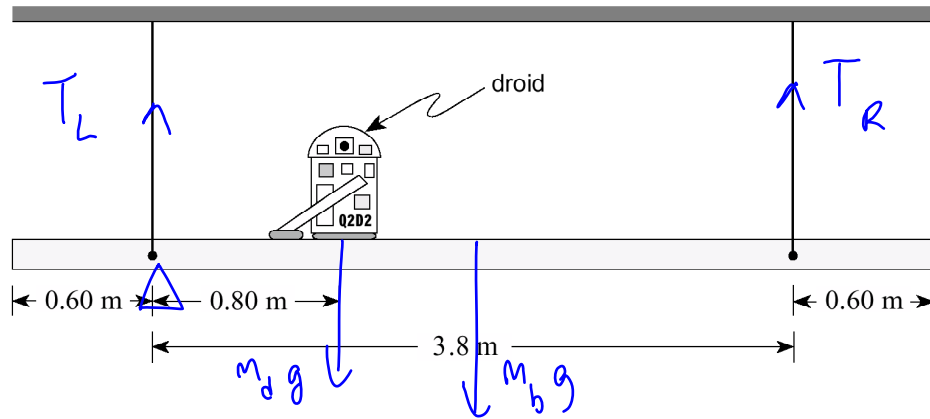
(3 marks)

$T = \frac{mg}{\cos \theta}$. As θ increases,
 $\cos \theta$ decreases
 $\therefore T$ increases.

9806

34.

A 25 kg droid rests on a 5.0 m long shelf supported by two cables as shown. The mass of the shelf is 12 kg.



Find the tension in **each** cable.

(7 marks)

$$\sum \tau_{ccw} = \sum \tau_{cw}$$

$$m_d g (.8) + m_b g (1.9) = T_R (3.8)$$

$$T_R = \frac{25(9.8)(.8) + 12(9.8)(1.9)}{3.8} = 110.4 = \boxed{1.1 \times 10^2 \text{ N}}$$

$$T_R + T_L = m_d g + m_b g$$

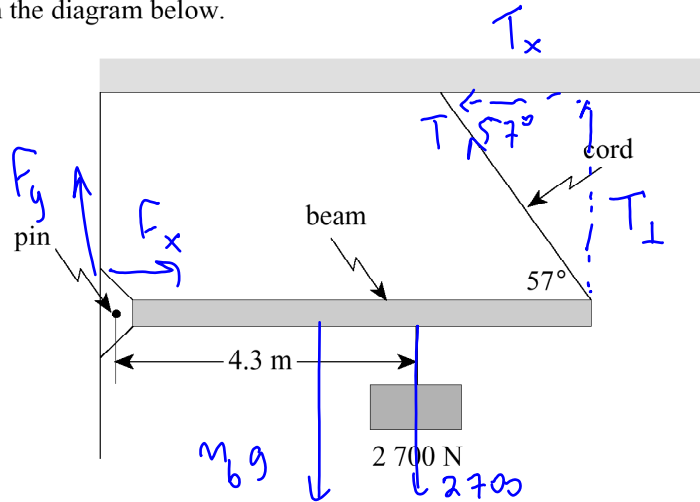
$$T_L = (25)(9.8) + 12(9.8) - 110.4 = \boxed{2.5 \times 10^2 \text{ N}}$$

Scholarship Questions. Nasty, but neat-o!!!!

9401

35.

A uniform beam of mass 75 kg and 6.4 m long supports a 2 700 N load and is held horizontal by a cord as shown in the diagram below.



- a) Draw a free-body diagram that identifies the forces acting on the beam. (3 marks)



- b) What are the magnitude and direction of the force that the pin exerts on the beam? (7 marks)

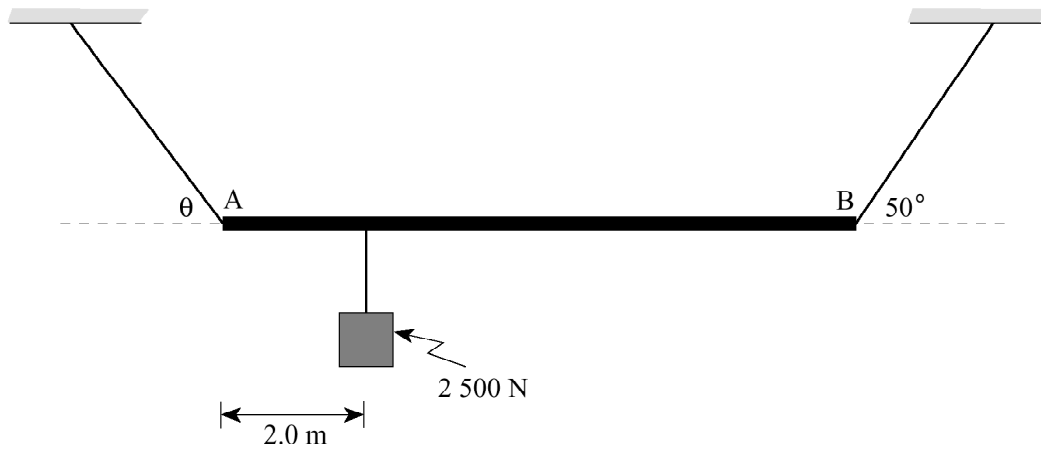
We need F_y and F_x !

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

9406

36.

A uniform horizontal beam AB, 8.0 m long and weighing 1 700 N is supported by cords attached as shown in the diagram below. A 2 500 N load is suspended from the beam 2.0 m from end A.

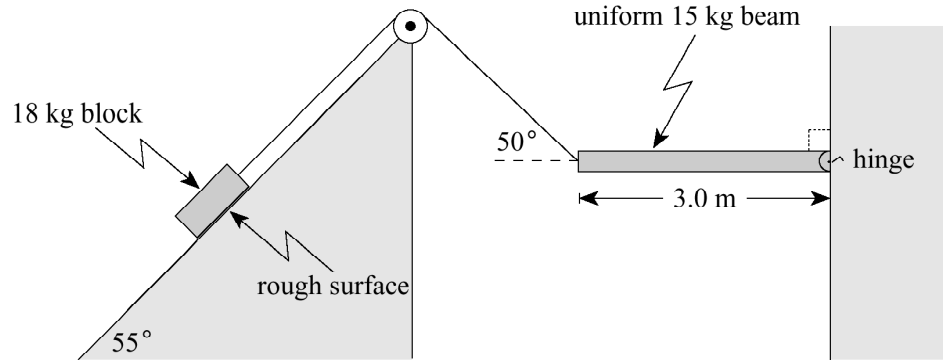


Determine the tension in the cord attached at end A and the angle θ that the cord makes with the horizontal. (diagram is not to scale) **(10 marks)**

9508

37.

The apparatus below is in equilibrium.



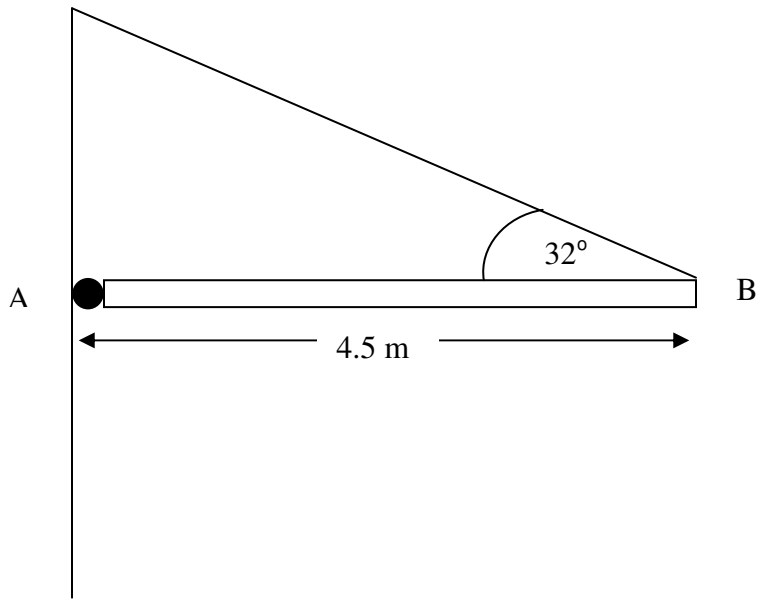
- a) Draw and label free body diagrams for **both** the block and the beam. **(4 marks)**
- b) What is the minimum coefficient of friction between the block and the inclined surface to prevent the block from sliding down the incline? **(8 marks)**

8501

38.

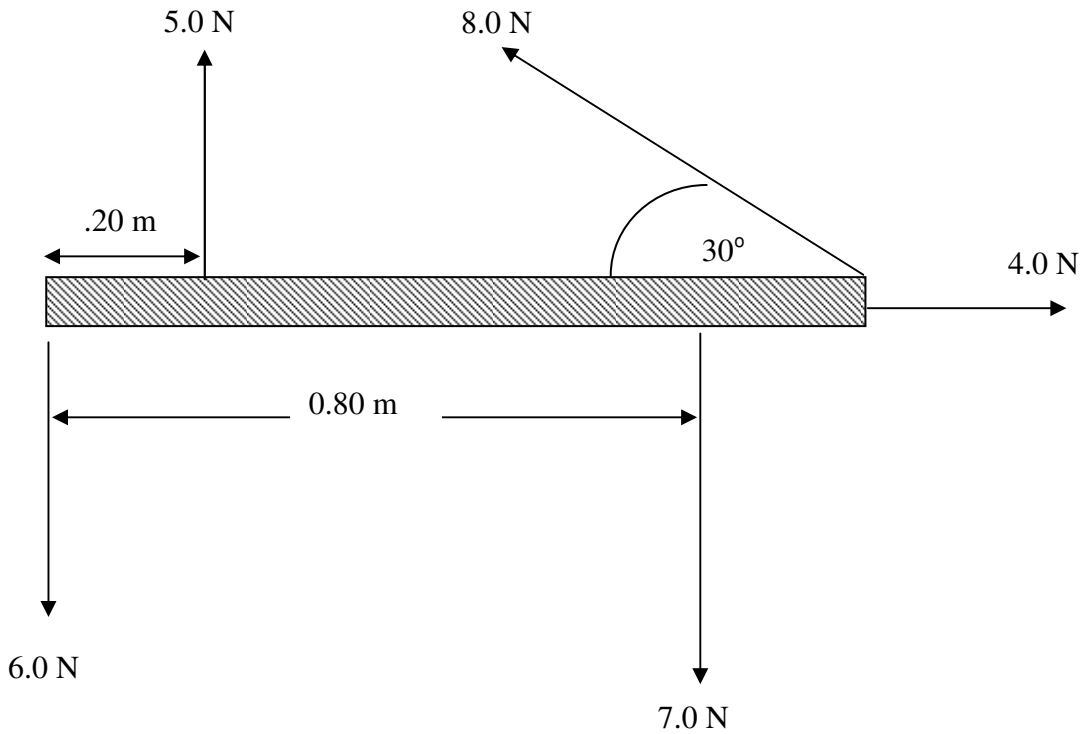
A uniform ladder has a mass of 16.0 kg and a length of 8.50 m. It stands on the ground and leans against a vertical wall, making an angle of 62.0 degrees with the ground. The friction between the wall and the ladder is not significant. What is the smallest force of friction between the ladder and the ground that will just prevent the ladder from slipping?

39.



The above diagram shows a uniform 24 kg beam hinged to the wall at A and supported at B by a cable. What is the magnitude of the **horizontal** force exerted by the wall on the beam? **(10 marks)**

0690
40.



A bar 1.0 m long has five forces acting on it as shown in the above diagram. What are the **magnitude, direction, and location** of the single force required to produce static equilibrium? The weight and thickness of the bar are insignificant. (12 marks)

Answers:

1. a
2. c
3. a
4. 82 N
5. d
6. c
7. c
8. 215 N
9. b
10. 3.8 kg
11. d
12. b
13. $F_B=1.49 \times 10^4 \text{ N}$,
 $F_A=3.31 \times 10^4 \text{ N}$
14. To balance the gravitational force on the sign, there must be an upwards force, since the sign is in equilibrium. In diagram B there is no upwards force, therefore diagram B is impossible.
15. b
16. d
17. d
18. 1600 N
19. a
20. a
21. c
22. $T_1=302 \text{ N}$, $T_2=224 \text{ N}$
23. b
24. c
25. c
26. $F_h=1.4 \times 10^3 \text{ N}$
27. d
28. c
29. a) $F_B=480 \text{ N}$, $F_A=350 \text{ N}$
b) increase c) see online solution key
30. a
31. d
32. 1500 N
33. a) $1.3 \times 10^2 \text{ N}$ b) increase
c) The vertical component of the tension is equal weight and is unchanged. Peter's horizontal force increases with a larger angle. The horizontal component of the tension is equal to Peter's and therefore also increases. Thus, the resultant tension is increased.
34. $T_L=253 \text{ N}$, $T_R=110 \text{ N}$
35. b) $F=1.9 \times 10^3 \text{ N}$ @ 42° above the horizontal
36. $T_A=3.0 \times 10^3 \text{ N}$, $\theta = 66^\circ$
37. $\mu = 0.48$
38. 41.7 N
39. 188 N
40. 4.96 N @ 54° N of E, located 0.15 m from the left side