1-Momentum

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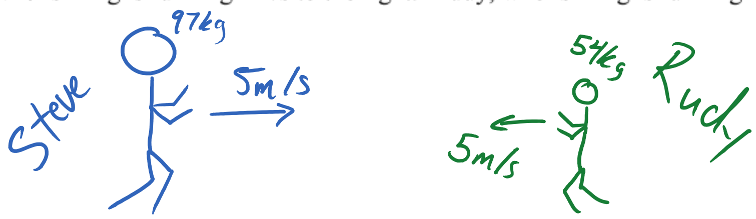
Momentum

Newton described momentum as the quantity of motion

Imagine a game of football:

Steve, who is 97kg is running 5m/s to the right. Rudy, who is 54kg is running 5m/s to the left.



When they collide for a tackle, who is "winning" in this situation?

Steve wins because he has more mass.

Is there a way that Rudy can end up winning?

Rudy can run Really fast!
↳ velocity

From this example we can see that an object's momentum depends on:

mass & velocity

Momentum Formula

$$\boxed{\vec{p} = m \times \vec{v}}$$

$N \cdot s = \frac{kg\,m}{s^{\cancel{2}}} \cdot s$

Momentum is a vector

Units: $\frac{kg\,m}{s}$ or Ns

Ex: A 6.2kg pumpkin is travelling 5.5m/s west. What is the momentum of this pumpkin?

$\vec{p} = m \cdot \vec{v}$

$\vec{p} = 6.2(5.5)$

$\vec{p} = 34.1\ \frac{kgm}{s}$ West

The pumpkin has 34.1Ns of momentum to the West.

Ex. a) A baseball has a mass of 0.14kg and is thrown at 22m/s to the east. Calculate the momentum of the baseball.

$m = 0.14kg$ ⊕ $\longrightarrow 22m/s$

$\vec{p} = \overset{m}{0.14} \cdot \overset{v}{(22)}$

$\vec{p} = 3.08\ Ns.$

The baseball has a momentum of 3.08Ns East.

b) What velocity would a 7.6kg bowling ball be going if it has the same amount of momentum as the baseball?

$\vec{p} = m \cdot \vec{v}$

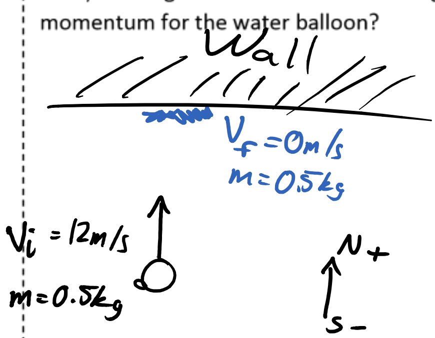
$\frac{3.08}{7.6} = \frac{7.6\ \vec{v}}{7.6}$

$\vec{v} = 0.41\ m/s$

The bowling ball will be going 0.41m/s East.

Ex. a) A 0.5 kg water balloon is thrown against a wall at 12m/s North. It breaks on impact. What is the ***change*** in momentum for the water balloon?

Δ → Final − initial



$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$\Delta\vec{p} = mv_f - mv_i$

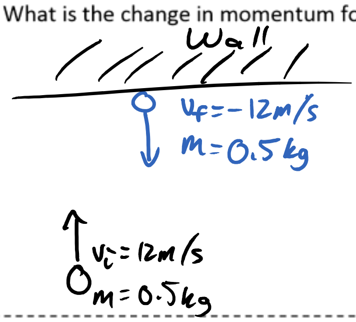
$\Delta\vec{p} = 0.5(0) - 0.5(12)$

$\Delta\vec{p} = 0 - 6$

$\Delta\vec{p} = -6\ kgm/s$

The change in momentum was $6\ \frac{kgm}{s}$ South.

b) A 0.5 kg bouncy ball is thrown against a wall at 12m/s North. It bounces back off of the wall, going 12m/s South. What is the change in momentum for the bouncy ball?



$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$\Delta\vec{p} = mv_f - mv_i$

$\Delta\vec{p} = 0.5(-12) - 0.5(12)$

$\Delta\vec{p} = -6 - 6$

$\Delta\vec{p} = -12\ kgm/s$

The change in momentum was $12\ \frac{kgm}{s}$ South.

Impulse

Impulse is defined as:

Change in momentum

We can derive the change in momentum from Newton's Second Law:

$\vec{p}_f - \vec{p}_i$

$\Delta\vec{p} = m v_f - m v_i$

$\Delta\vec{p} = m(v_f - v_i)$

$\Delta\vec{p} = m\Delta\vec{v}$

↑ symbol for impulse

$F = ma \quad a = \left(\frac{\Delta v}{t}\right)$

$t \times F = m \cdot \frac{\Delta v}{\cancel{t}} \cancel{\times t}$

$F \times t = \underbrace{m\Delta v}_{\text{Impulse}}$

$$\boxed{\begin{gathered}\Delta\vec{p} = \vec{p}_f - \vec{p}_i \\ \text{or} \\ \Delta\vec{p} = \vec{F} \times t\end{gathered}}$$

Since force and momentum are both ___vectors___, impulse is also a ___vector___ and direction is important.

Example: A baseball bat applies a 50N force (in the positive direction) on a baseball that has 250g of mass for a time period of 0.10s.

F $\quad m = 0.250\,kg \quad t$

a) What is the impulse acting on the ball?

$\Delta\vec{p} = F \times t$

$\Delta\vec{p} = 50 \times 0.10$

$\Delta\vec{p} = 5\ \frac{kg\,m}{s}$

The impulse on the ball was $5\ \frac{kg\,m}{s}$ in the positive direction.

b) If the ball had an initial velocity of -5m/s, determine the velocity of the ball after being hit by the bat.

$v_i \quad v_f$

$\Delta\vec{p} = m v_f - m v_i$

$5 = 0.250 v_f - 0.250(-5)$

$5 = 0.250 v_f + 1.25$

$-1.25 \qquad -1.25$

$\frac{3.75}{0.250} = \frac{\cancel{0.250} v_f}{\cancel{0.250}}$

$v_f = 15\,m/s$

The ball flies at +15 m/s after it was hit.

Example: A soccer player shot a 0.450kg ball at 13m/s East. The goal keeper applied a 135N force to stop the ball.

$-$ v_i $+$ F v_f m

a) What direction did the keeper force the ball?

West

West $\longleftarrow$ East

b) How long did the keeper's hands contact the ball before it was stopped?

t

$$\boxed{\begin{aligned}\Delta\vec{p} &= mv_f - mv_i\\ \Delta\vec{p} &= F\cdot t\end{aligned}}$$

$$F\cdot t = mv_f - mv_i$$

$$135\cdot t = 0.450(0) - 0.450(-13)$$

$$\frac{135\cdot t}{135} = \frac{0+5.85}{135} \Rightarrow t = 0.04s$$

The keeper forced the ball for 0.04s.

Explain, using the principles of momentum, why an athlete will use a "follow through" when striking a ball (for example: in soccer, golf, tennis, etc.).

$$\Delta p = F\cdot t$$

Conservation of Momentum and Collisions

Law of Conservation of Momentum: Momentum can neither be created nor destroyed, only transferred from one object to another.

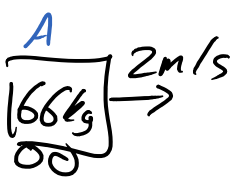
Identifying

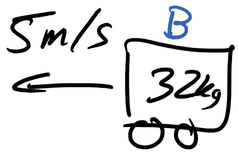
Conservation Problems	**vs.**	**Impulse Problems**
• Multiple objects • Closed system ↳ No outside forces *Collisions or explosions*		• Only one important object. • Outside forces are acting on the object.

To find the momentum of a system you Find the momentum of each piece then add.

Example: A 32kg cart is heading west at 5m/s and a 66kg cart is heading east at 2m/s. What is the momentum of the system?

W ⟵⟶ E
− +





$p_A = m_A v_A$

$p_A = 66 \cdot (2)$

$p_A = 132\,Ns$

$p_B = m_B v_B$

$p_B = 32(-5)$

$p_B = -160\,Ns$

$\vec{P}_{Total} = p_A + p_B = 132 + (-160)$

$\vec{P}_{Total} = -28\,Ns$

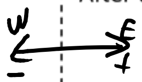
Conservation of Momentum means that the Total initial amount of momentum in a closed system does not change, even after an event or collision.

To summarize, this means: Total Initial Momentum = Total Final Momentum

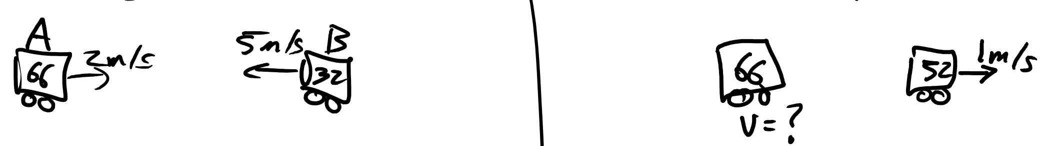
Or as a formula:

$$\vec{P}_{Ti} = \vec{P}_{Tf}$$

Example: The same carts from the previous example (32kg heading 5m/s west and 66kg heading 2m/s east) collide. After the collision the 32kg mass rebounds and travels 1m/s east. Determine the final velocity of the 66kg mass.



Before | After



$$P_{Ti} = P_{Tf}$$

$$P_{Ai} + P_{Bi} = P_{Af} + P_{Bf}$$

$$m_A v_i + m_B v_i = m_A v_{fA} + m_B v_{fB}$$

$$66(2) + 32(-5) = 66 v_{fA} + 32(1)$$

$$132 + (-160) = 66 v_{fA} + 32$$

$$-28 = 66 v_{fA} + 32$$

$$-32 \qquad\qquad -32$$

$$\frac{-60}{66} = \frac{\cancel{66} v_{fA}}{\cancel{66}}$$

$$v_{fA} = -0.91 \text{ m/s}$$

The 66kg cart is going 0.91m/s West.

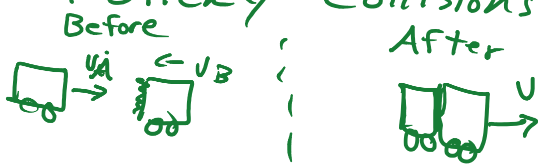
Elastic collisions vs. **Inelastic collisions**

Bouncy collisions

Like this example

The final velocities will likely be different.

Sticky collisions



They end with the same velocity.

$$m_A v_{Ai} + m_B v_{Bi} = m_A v + m_B v$$

or

$$m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B) v$$

*Explosions are reverse sticky collisions.

Momentum Connecting with Forces and Kinematics

From our two momentum formulae we can identify the parts that are also used in Force and Kinematics calculations:

$$p = mv \qquad \Delta p = F_{net} \cdot t$$

Kinematics

Forces

If we know the initial or final conditions we can use that information to learn details about the rest of the situation.

Example: A 3kg book has a 38N force applied upward on it for 2s. How high will the book go after the applied force stops?

Pushing the book | AND THEN | Book Free Fall (projectile motion)

$F_A = 38N$

3 kg

$F_g = 3(9.8) = 29.4N$

$t = 2s$

$v_f \rightarrow v_i$

$v = 0m/s$ $d = ?$

d

$v_i = 5.73m/s$

$v_f = 0m/s$

$a = -9.8m/s^2$

$d = ?$

$t =$

$F_{Net} = F_A - F_g = 38 - 29.4 = 8.6N$

$F_{Net} \cdot t = mv_f - mv_i$

$8.6 \times 2 = 3v_f - 3(0)$

$\frac{17.2}{3} = \frac{3v_f}{3}$

$5.73 = v_f$

$v_f^2 = v_i^2 + 2ad$

$0^2 = 5.73^2 + 2(-9.8)d$

$0 = 32.8 - 19.6d$

$\frac{19.6d}{19.6} = \frac{32.8}{19.6}$

$d = 1.7m$

After leaving the hand the book rose 1.7m.

Example: A 5kg box is sitting on a table with a coefficient of friction of 0.22. A 0.30kg sticky ball is thrown at the box at a velocity of 12m/s. When the ball hits the box, they stick together. How far will the box slide across the table after the collision?

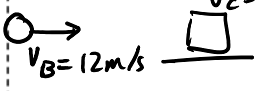
Collision
(Law of Cons v. of Mom.)

AND THEN

Before

$m_B = 0.3kg$ $m_c = 5kg$

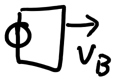
$v_c = 0\,m/s$



$v_B = 12m/s$

After

$m_{BC} = 5.3kg$



v_{BC}

$m_B v_B + m_c v_c = m_{BC} v_{BC}$

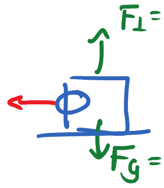
$0.3(12) + 5(0) = 5.3\, v_{BC}$

$\frac{3.6 + 0}{5.3} = \frac{5.3 v_{BC}}{5.3}$

$v_{BC} = 0.679 m/s$

The ball and box slid 10.7cm

Sliding w/ Friction
(Forces & kinematics)



F_f

$F_\perp = 51.94N$

$F_g = 51.94N$

$F_g = m \cdot g$

$F_g = (5+0.3)(9.8)$

$= 51.94N$

$F_f = \mu F_\perp$

$F_f = 0.22(51.94)$

$F_f = 11.428N$

$F_{net} = F_f$

$m \cdot a$

$5.3 \times a = 11.428$

$a =$

$v_i = 0.679 m/s$

$v_f = 0 m/s$

$a = -2.156 m/s^2$

$d = ?$

$t =$

$v_f^2 = v_i^2 + 2ad$

$0^2 = 0.679^2 + 2(-2.156)d$

$0 = 0.461 - 4.312d$

$d = 0.107m$