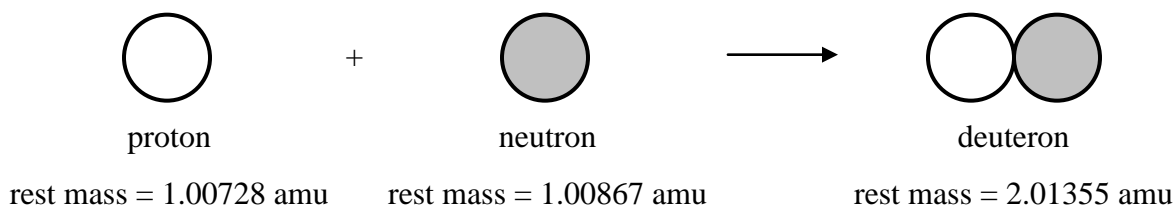


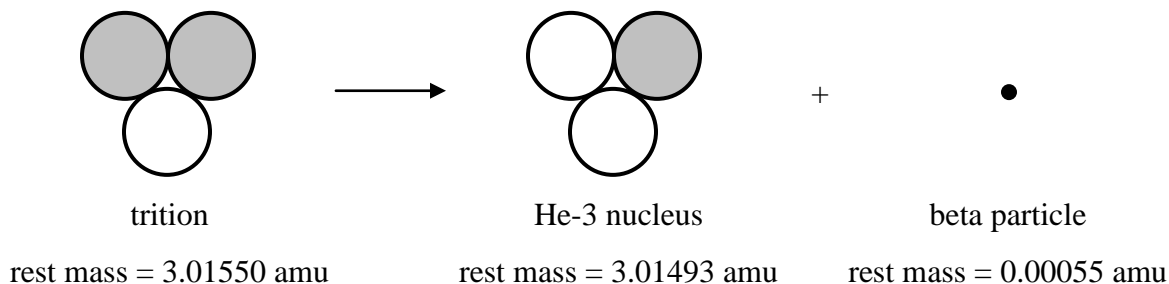
Why are fission reactors energetically so economical?

**The Model: Mass Defect and Nuclear Binding Energy**

Most of the elements have more than one isotope. Hydrogen has two naturally-occurring, non-radioactive isotopes: H-1 (“protium”) and H-2 (“deuterium”). The deuteron is composed of a proton and a neutron, which are held together by the strong nuclear force.



H-3 (“tritium”) is the third isotope of hydrogen. Tritium is radioactive and undergoes beta decay:



Helium-3 is not radioactive.

The atomic mass unit (amu) is equivalent to  $1.66054 \times 10^{-27}$  kg. Einstein determined that the rest mass energy of a particle is

$$E = mc^2$$

where  $c$  is the speed of light ( $2.998 \times 10^8$  m/s).

**Key Questions**

1. Consider the neutron decay of deuterium (eq 1).



- a. What is the rest mass (in kg, with six significant figures) of the reactant of eq 1?
- b. What is the sum of rest masses (in kg, with six significant figures) of the products of eq 1?

- c. The **mass defect** ( $\Delta m$ ) for a nuclear reaction is simply the sum of the rest masses of the products minus the sum of the rest masses of the reactants.

$$\Delta m = \sum_p m_{\text{products}} - \sum_r m_{\text{reactants}}$$

What is the mass defect (in kg) for the neutron decay of deuterium (eq 1)?

- d. If  $E = mc^2$ , then the **nuclear binding energy** ( $\Delta E$ ) of a nucleus is the square of the speed of light ( $c^2$ ) times the difference between the masses of all nucleons and the mass of the nucleus ( $\Delta m$ ).

$$\Delta E = (\Delta m)c^2$$

(Recall that  $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$ .) What is the nuclear binding energy of deuterium (in joules per deuteron)?

2. Consider the beta decay of tritium (eq 2).



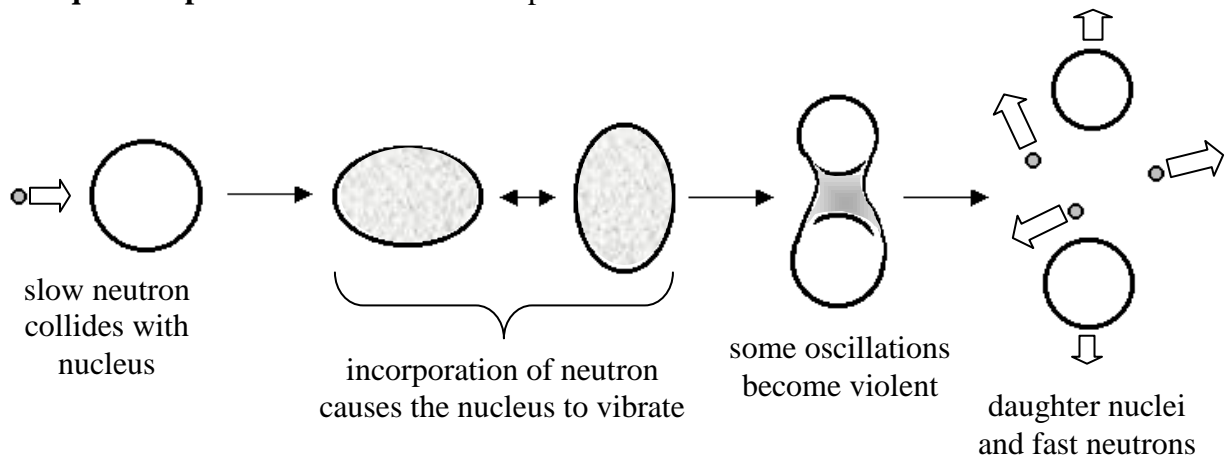
As you did in Question 1 for the neutron decay of H-2, determine the following for the beta decay of H-3: (a) the rest mass of the reactant (in kg, with six significant figures); (b) the sum of rest masses of the products (in kg, with six significant figures); (c) the mass defect; and (d) the nuclear binding energy of tritium (in joules per tritium).

3. You probably remember the First Law of Thermodynamics as: “Energy can neither be created nor destroyed, but it may be converted from one form to another.” What does it mean for tritium to have a “negative nuclear binding energy”? Where does the “missing” energy go? (*Hint*: Consider what *type* of masses you subtracted when you calculated the mass defect.)
4. Why is H-3 radioactive while H-2 is not?

### The Model: Nuclear Fission

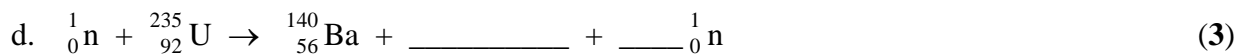
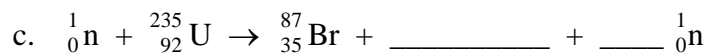
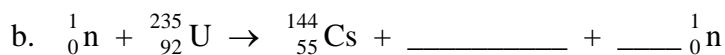
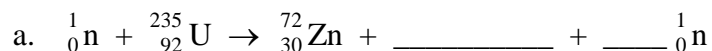
Fission reactors employ Uranium-235 as “fuel”. A slow neutron collides with and is absorbed by a U-235 nucleus, becoming U-236. The “excited” U-236 nucleus then undergoes spontaneous fission. The frequently-observed daughter nuclei include Zn-72, Br-87, Kr-93, Rb-90, Cs-144, Ba-140, La-146, and Sm-160. By-products of each fission are fast neutrons. The fuel rods are submerged in water, which “moderates” the fast neutrons. The water converts fast neutrons into slow neutrons, which permits more U-235 nuclei to become U-236, leading to more fission.

The **liquid-drop model** of the nucleus helps us understand nuclear fission.



### Key Questions

5. Use the information provided above in the Model to balance each of the following nuclear equations that describe the spontaneous fission that occurs after a slow neutron collides with a U-235 nucleus.



6. Consider the nuclear reaction in eq 3 (*i.e.*, the reaction of Question 5d). The isotopic masses of U-235, Ba-140, and Kr-93 are 235.04392 amu, 139.91058 amu and 92.93113 amu, respectively.

a. What are the sums of rest masses (in kg, with 6 sig figs) of reactants and products in eq 3?

b. What is the mass defect (in kg) for the absorption of a neutron by U-235 followed by spontaneous fission?

c. How much energy (in kJ) is released *per mole* of U-235 that is consumed as fuel? (Yes, nuclear fission reactors have their problems – the build-up of radioactive “left-over” by-products of the fission reactions, plus the possibility of going critical. But to understand why fission reactors are economical to use, compare the amount of energy you just calculated to the  $\Delta G^\circ$ 's of -5300 kJ/mol for the combustion of octane and -474 kJ/mol for the cell reaction in the hydrogen-oxygen fuel cell.)